JMEPEG (2008) 17:888–893
DOI: 10.1007/s11665-008-9225-5

Correlation of Yield Strength and Tensile Strength with Hardness for Steels

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(Submitted February 1, 2008)

Hardness values as well as yield and tensile strength values were compiled for over 150 nonaustenitic, hypoeutectoid steels having a wide range of compositions and a variety of microstructures. The microstructures include ferrite, pearlite, martensite, bainite, and complex multiphase structures. The yield strength of the steels ranged from approximately 300 MPa to over 1700 MPa. Tensile strength varied over the range of 450-2350 MPa. Regression analysis was used to determine the correlation of the yield strength and the tensile strength to the diamond pyramid hardness values for these steels. Both the yield strength and tensile strength of the steels exhibited a linear correlation with the hardness over the entire range of strength values. Empirical relationships are provided that enable the estimation of strength from a bulk hardness measurement. A weak effect of strain-hardening potential on the hardness-yield strength relationship was also observed.

Keywords hardness testing, steels, tensile testing

1. Introduction

The relation between flow strength and hardness has been theoretically determined to be

$$H = cS \qquad \text{(Eq 1)}$$

where S is the uniaxial flow strength and H is hardness. The factor c is termed as elastic constraint factor and has a value of approximately 3 for metals that do not strain harden appreciably when H is measured in $\text{kg}_\text{f}/\text{mm}^2$ and S is measured in MPa (Ref 1-3). The flow strength value in Eq 1 corresponds to the plastic strain that is unique to the hardness test performed, or more specifically, to the geometry of the indenter tip. In the case of diamond pyramid hardness (DPH), the flow stress corresponds to a plastic strain of 0.08 (Ref 4).

Cahoon et al. (Ref 5, 6) offered expressions relating hardness and tensile strength and yield strength in the form of

$$\text{TS} = \left(\frac{H}{2.9}\right)\left(\frac{n}{0.217}\right)^n \qquad \text{(Eq 2)}$$

$$\text{YS} = \left(\frac{H}{3}\right)(0.1)^n \qquad \text{(Eq 3)}$$

where TS and YS are tensile strength and yield strength, respectively, and n is the strain-hardening exponent. These expressions show excellent agreement (<2%) in calculating the tensile properties of a ferritic steel at temperatures up to 400 °C (Ref 7). Use of Cahoon's expressions requires prior knowledge of the strain-hardening exponent either directly from uniaxial tensile tests or indirectly through Meyers index or empirical methods (Ref 8).

In the present investigation, room temperature hardness and strength values were compiled from 20 years of thesis work at the Advanced Steel Processing and Products Research Center at the Colorado School of Mines (Ref 9-28). Since the strength and hardness of the steels covered such a large range, all hardness values were converted to diamond pyramid hardness, also known as Vickers hardness, in accordance with ASTM E140-05 (Ref 29). A majority of hardness values were converted from the Rockwell B or Rockwell C scales. The objective of the present study is to provide correlations to estimate the yield strength and tensile strength of steel based upon a bulk hardness measurement.

2. Results

2.1 Full Data Set Correlations

Figure 1 shows all of the compiled strength-hardness data. Yield strength shows a clear linear relationship with the diamond pyramid hardness for the entire strength range (Fig. 1a). A least-squares linear regression gives the correlation for yield strength as

$$\text{YS} = -90.7 + 2.876H_\text{V} \qquad \text{(Eq 4)}$$

where yield strength has units of MPa and H_V is diamond pyramid hardness which uses traditional units ($\text{kg}_\text{f}/\text{mm}^2$). Regression analysis indicates that Eq 4 has a coefficient of determination, R^2, of 0.9212 and a standard error of 102 MPa, which indicates that over the hardness range examined, steel yield strength is linearly correlated with hardness. It may be expected the constant in Eq 4 to be zero since a steel with zero hardness should also have zero strength. However, the standard error associated with the constant in Eq 4 is 22.3 MPa, which indicates that the nonzero value for the

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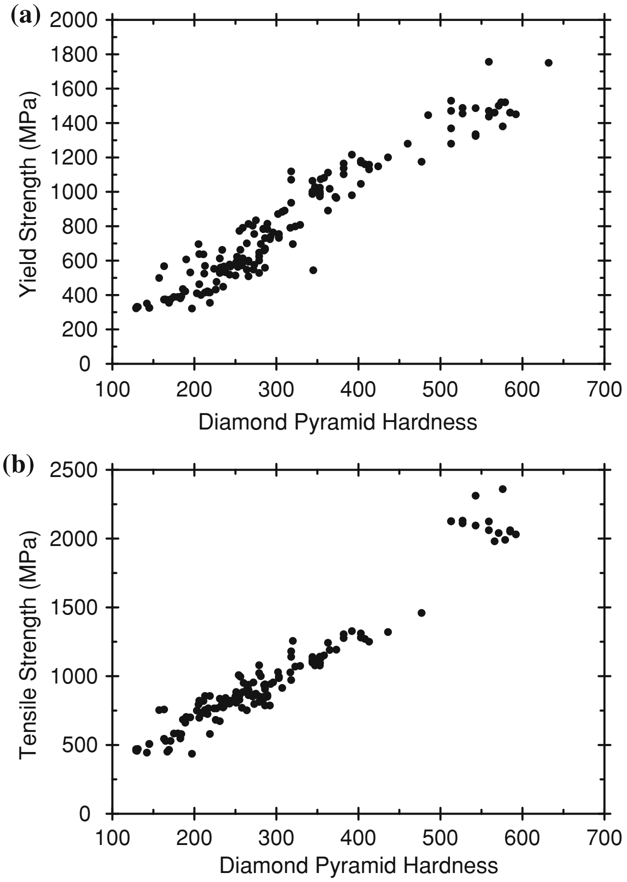

Fig. 1 Plot of (a) yield strength and (b) tensile strength of various steels as a function of hardness. Data from Ref 9-28

constant is correct and that the yield strength-hardness correlation would be expected to be nonlinear at hardness values less than 130 DPH.

Tensile strength also has a linear relationship with the diamond pyramid hardness over the entire strength range (Fig. 1b). A least-squares linear regression gives the correlation for tensile strength as

$$\mathrm{TS} = -99.8 + 3.734 H_{\mathrm{V}} \qquad \text{(Eq 5)}$$

where tensile strength has units of MPa. Equation 5 has an R^2 value of 0.9347 and a standard error of 112 MPa. The standard error associated with the constant of Eq 5 is 24.5 MPa, which indicates that over the hardness range examined, tensile strength has a linear correlation with hardness, with the correlation becoming nonlinear at DPH values less than 130.

Figure 2 shows results of the regression analysis with the standard error shown, while Fig. 3 illustrates the difference between the predicted strength value (i.e., Eq 4 and 5) and the actual strength value. The yield strength-hardness regression, Eq 4, shows no systematic deviations over the strength and hardness range examined. However, the tensile strength-hardness regression, Eq 5, appears to slightly under predict strength at higher hardness values. This slight deviation from randomness indicates that a linear regression may not be ideal for the tensile strength-hardness data.

Tensile strength is often cited to vary linearly with the Brinell hardness, H_{B}, as

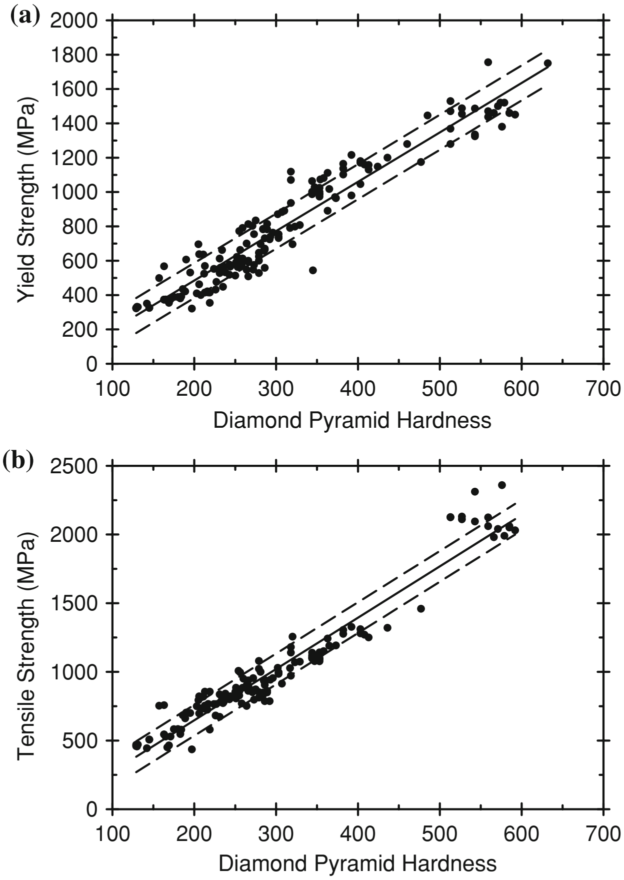

Fig. 2 Plot of (a) yield strength and (b) tensile strength of various steels as a function of hardness. Solid line represents results of linear regression analysis from Eq 4 and 5. Dashed lines represent ±1 standard error of the regression analysis. Data from Ref 9-28

$$\mathrm{TS} = 3.45 H_{\mathrm{B}} \qquad \text{(Eq 6)}$$

for tensile strength in units of MPa (Ref 30). To generate a curve for Eq 6, diamond pyramid hardness values between 100 and 650 were converted to Brinell hardness according to ASTM E140-05 (Ref 29) and the tensile strength was calculated. Figure 4(a) shows Eq 6 overlaid on the tensile strength-hardness data. Figure 4(b) illustrates the difference in the predicted tensile strength using Eq 6 and the actual tensile strength value. At higher hardness values, Eq 6 tends to under predict the tensile strength. The effect is significant for the highest strength steels evaluated in this study.

2.2 Correlations Sorted by Microstructure

Figure 5 shows the hardness-strength data grouped by microstructure. Microstructures were divided into three categories: (1) martensitic microstructures, which include as-quenched and quenched and tempered martensite, (2) nonmartensitic microstructures, which include ferrite/pearlite, bainite, and acicular ferrite, and (3) complex phase microstructures, which are those that consist of mixtures of ferrite and other phases such as bainite, martensite, or retained austenite.

Correlations for these three microstructural groups were performed and the values for the constants, coefficients, and R^2 values appear in Table 1. The correlations for the martensitic microstructures are slightly better than for the nonmartensitic

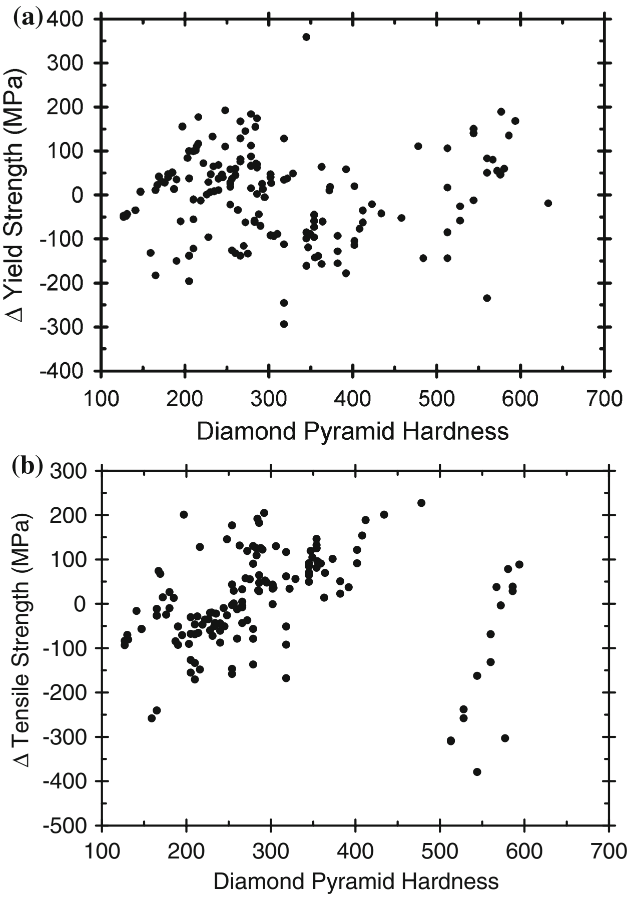

Fig. 3 Plot of the difference in the predicted strength value minus the actual strength value for (a) yield strength, Eq 4, and (b) tensile strength, Eq 5. Data from Ref 9-28

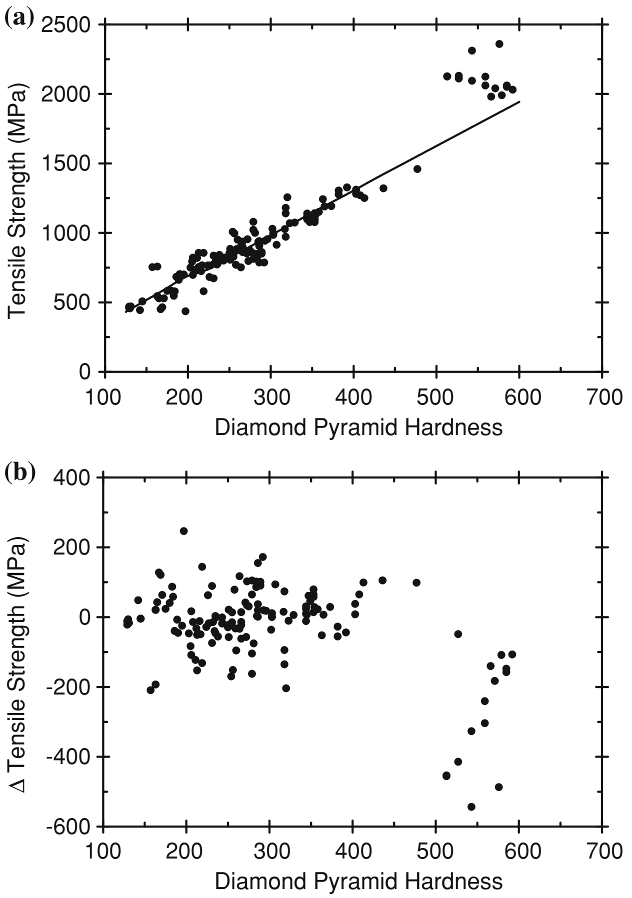

Fig. 4 (a) Plot of Eq 6 overlaid on the tensile strength. (b) Plot of the difference in the predicted tensile strength value calculated from Eq 6 minus the actual tensile strength value. Data from Ref 9-28

microstructures. The correlation for the complex phases is good for the tensile strength but not very good for the yield strength. Because of the small number of points in the complex phase data set, the reliability of these two correlations is somewhat less as compared to the other two microstructural groups.

2.3 Correlations Sorted by Tensile to Yield Strength Ratio

Strain-hardening potential is a measure of the maximum amount of hardening that a steel can achieve during plastic deformation. Plastic instability will limit the amount of strain hardening that can be obtained. In a tensile test the tensile strength corresponds to the point of tensile instability for the steel. A measure of strain-hardening potential is the ratio of the tensile strength to the yield strength (TS:YS). With the use of this measure, materials that exhibit a large strain-hardening potential will have a high TS:YS ratio. The hardness-strength data were divided into three groups: (1) low TS:YS (TS:YS ≤ 1.23), (2) medium TS:YS ($1.23 < \text{TS:YS} < 1.56$), and (3) high TS:YS (TS:YS ≥ 1.56). Figure 6 illustrates the effect of strain-hardening potential on the strength-hardness relationship. The values for the constants, coefficients, and R^2 values are given in Table 1. Generally, for a given hardness the predicted strength will be greater for steels with low values of TS:YS (i.e., low strain-hardening potential) as compared to steels with a high value of TS:YS (i.e., high strain-hardening potential).

2.4 Correlation with Average Strength

The size of the standard error (~100 MPa) associated with Eq 4 and 5 suggests that yield strength and tensile strength do not solely correlate with hardness and that other material properties such as strain-hardening exponent may factor into the correlation. Figure 6 also indicates that for a given hardness, yield strength weakly depends on the tensile strength, or more accurately, the ratio of the tensile strength to the yield strength. With this in mind, an average strength was defined as the average of the yield strength and tensile strength for a given steel. Figure 7 shows the average strength plotted against the hardness. A least-squares linear regression gives the correlation for average strength as

$$S_{\text{ave}} = -93.8 + 3.295 H_{\text{V}} \qquad \text{(Eq 7)}$$

where average strength, S_{ave}, has units of MPa. Equation 7 has an R^2-value of 0.9610 and a standard error of 75 MPa. The standard error associated with the constant in Eq 7 is 16.5 MPa. The correlation for average strength is a better correlation than those for yield and tensile strength, Eq 4 and 5, respectively, even though the practical value of Eq 7 is less.

3. Discussion

Table 1 shows the results of a least-squares linear regression performed for the different data sets where strength, S, has the form

$$S = a_0 + a_1 H_V \quad \text{(Eq 8)}$$

where a_0 and a_1 are regression constant and regression coefficient, respectively, for a given data set. In Eq 8 strength has units of MPa and H_V is a diamond pyramid hardness value. Table 1 also indicates the hardness range over which each correlation is valid. From a practical standpoint, the standard error (~100 MPa) associated with the linear regressions makes them less than perfect for estimating strength from a bulk hardness measurement. However, the analysis does indicate that both yield strength and tensile strength have a linear correlation with hardness over a very large range of hardness and strength that result from a wide variety of steel chemistries and processing methods. In addition, the analysis indicates that both yield strength and tensile strength are coupled within a bulk hardness measurement, with the hardness measurement depending on the strain-hardening behavior as well as the strength of the material.

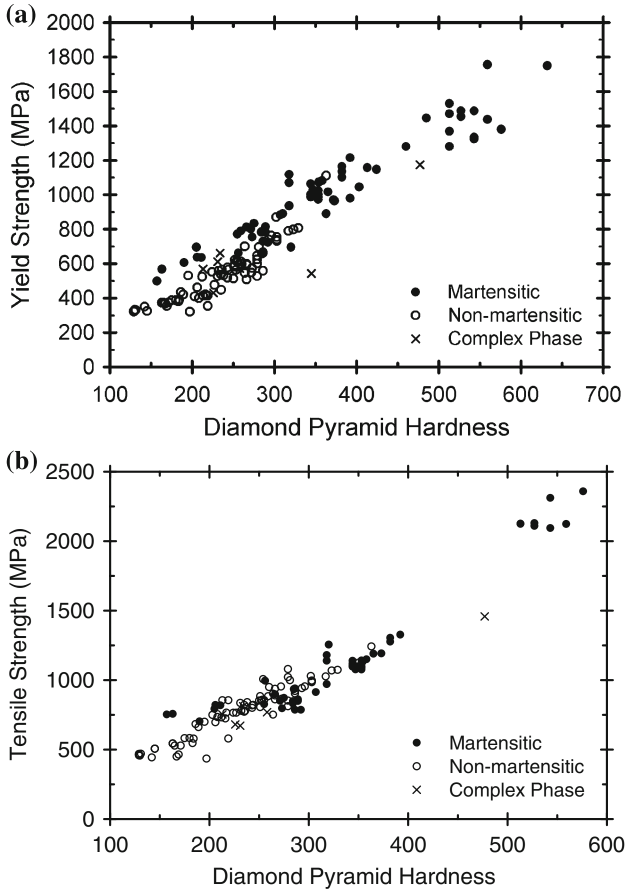

Fig. 5 Plot of (a) yield strength and (b) tensile strength of various steels as a function of hardness. Data is grouped by microstructure. Data from Ref 9-28

Empirical correlations for the Medium TS:YS data group are best suited for estimating strength from a hardness value for steels. The correlations for this group have the highest coefficient of determination for both yield and tensile strength and cover almost the entire hardness range examined in this study. Steels that exhibit extremes in strain-hardening potential, either low or high, will fall outside of the estimation when using correlations for the Medium TS:YS data group. However, since this data group covers the largest number of different steels when grouped by TS:YS, it is expected to give reasonable estimations, particularly when minimal information about the steel is known. Determining strength from hardness may be important when using small test samples when full-size or sub-size uniaxial tensile specimens are not feasible and a strength estimate is required. Small test samples may be encountered during heat treatment studies and thermomechanical simulations.

Interestingly, the ratio of Eq 5 to 4 suggests that strain-hardening potential for a steel decreases with hardness. Figure 8 shows the TS:YS values for all steels in this study along with the predicted value from the ratio of Eq 5 to 4. The average value of TS:YS can be considered the maximum strength increase due to strain hardening that can occur up to the point of instability for a given steel with the strength increase being a fraction of the yield strength of the material.

Table 1 Table of regression analysis results for all strength-hardness data sets

Data set	Strength value	Regression constant	Regression coefficient	R^2	Valid range (DPH)	Data points
All data	Yield	−90.7	2.876	0.9212	129–632	165
All data	Tensile	−99.8	3.734	0.9347	129–592	159
All data	Average	−93.8	3.295	0.9610	129–592	144
Martensitic	Yield	110.9	2.507	0.9088	157–632	66
Martensitic	Tensile	−273.6	4.279	0.9017	157–576	50
Non-martensitic	Yield	−84.8	2.646	0.8414	129–363	75
Non-martensitic	Tensile	2.5	3.339	0.8910	129–363	75
Complex phases	Yield	55.1	2.105	0.7045	213–477	7
Complex phases	Tensile	54.0	2.969	0.9610	213–477	7
Low TS:YS	Yield	15.3	2.860	0.8794	195–436	44
Low TS:YS	Tensile	10.9	3.150	0.8716	195–436	44
Medium TS:YS	Yield	−70.5	2.736	0.9719	129–592	69
Medium TS:YS	Tensile	−99.8	3.800	0.9620	129–592	69
High TS:YS	Yield	−94.8	2.466	0.9335	145–576	31
High TS:YS	Tensile	−205.7	4.410	0.9630	145–576	31

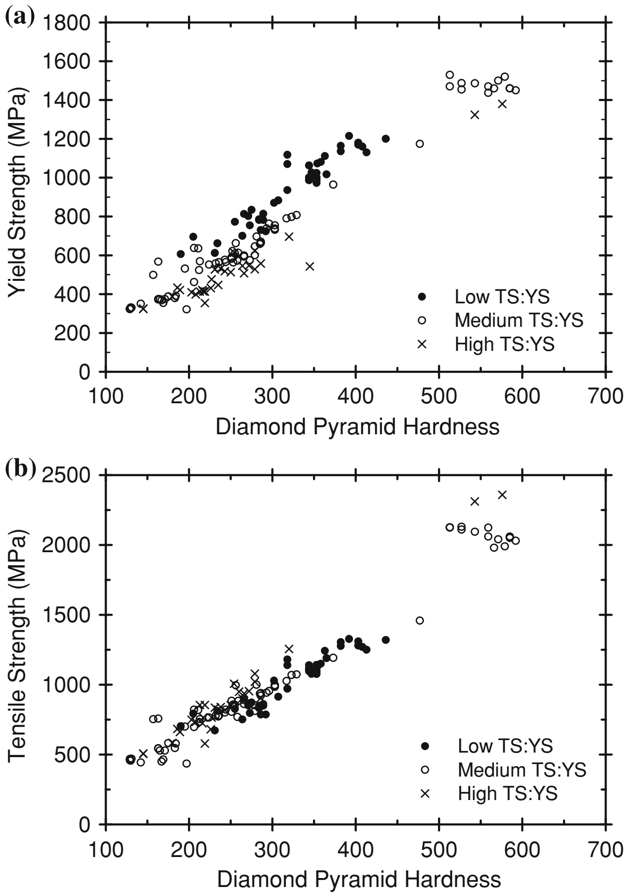

Fig. 6 Plot of (a) yield strength and (b) tensile strength of various steels as a function of hardness. Data is grouped by strain-hardening potential, TS:YS. Data from Ref 9-28

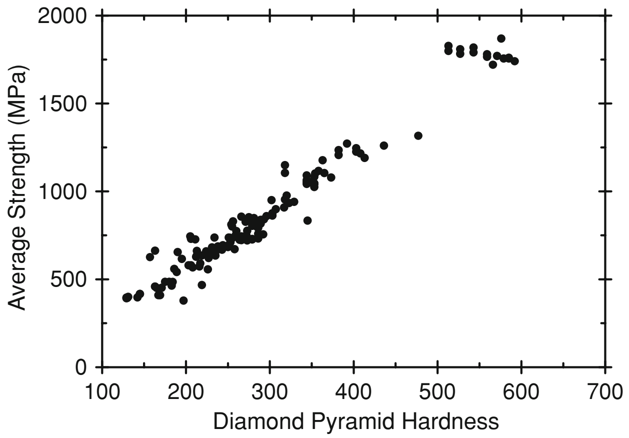

Fig. 7 Plot of the average strength of various steels as a function of hardness. Data from Ref 9-28

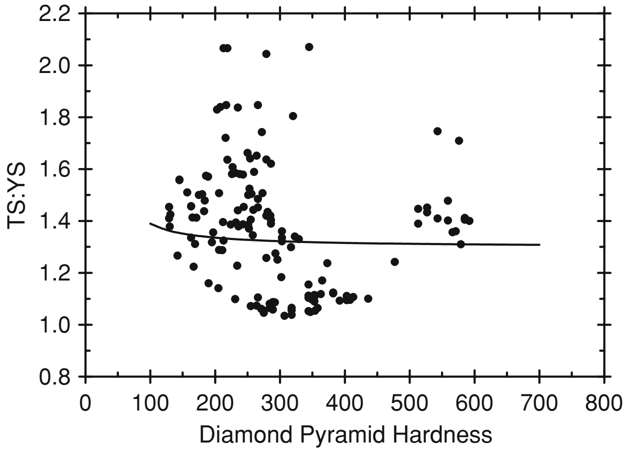

Fig. 8 Plot of the tensile strength to yield strength ratio, TS:YS. The ratio of Eq 5 to Eq 4 is also indicated. Data from Ref 9-28

The average value of TS:YS for the steels in this study is 1.33. The average TS:YS value of the martensitic steels and non-martensitic steels are 1.21 and 1.49, respectively.

Since the plastic strain associated with DPH is 0.08, it is not too surprising that the average strength correlation is very good over a large strength range and for the large number of data points used in the correlation. While uniform elongations were not compiled for the steels in this study, a plastic strain of 0.08 is expected to fall below the uniform elongation for most steels in this study and that the average of the yield and tensile strengths would approximate the flow strength at a plastic strain 0.08.

Force equilibrium suggests that flow strength should vary directly with hardness according to Eq 1 and a metal with zero hardness should have zero strength. However, the non-zero constant in empirically derived relationships between strength and hardness is not unexpected. Previous relationships for magnesium alloys and aluminum alloys also contain a non-zero constant in the regression although the strength range of the correlation is much narrower than the current study (Ref 31-33). The value of the regression constant may be attributed to the material and indenter constant term

$$cS_0 \sinh^{-1}(\beta e) \qquad \text{(Eq 9)}$$

which can be added to the right hand side of Eq 1 (Ref 4). The variables S_0 and β are material constants and e corresponds to the plastic strain from the indentation while c is the elastic constrain factor. For the data set used in this study it is apparent that Eq 5 is more appropriate for estimating tensile strength from a bulk hardness measurement as compared to Eq 6. Equation 5 requires that a nonzero constant be used for the linear tensile strength-hardness correlation over a strength or hardness range that is typical of steels.

4. Summary

Yield strength and tensile strength were found to have a linear correlation with hardness for steels with yield strengths of 325 MPa to over 1700 MPa and tensile strengths between 450 and 2350 MPa. For steels that can exhibit a large amount of strain hardening, the correlations predict a lower strength for a given hardness. Results indicate that strength varies linearly over a hardness range that is typical of nonaustenitic hypoeutectoid steels. At low hardness or strength levels, it is expected that strength has a nonlinear correlation with hardness.

Acknowledgment

The authors gratefully acknowledge B.S. Levy and K.D. Clarke for helpful discussions about the data analysis.

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